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Reflections from Rotary-Vane Precision Attenuators

Due to finite thickness of the absorbing vanes, the scattering coefficients [1] S_{11} and S_{22} of a rotary-vane attenuator [2] are not zero (as would be true of an "ideal" device) but are, in fact, functions of attenuator setting. Thus, terminal reflections cannot be eliminated by simply fixed-tuning the ports. In this correspondence we derive an equation relating S_{11} (S_{22}) to the attenuator setting by considering the effects of small reflections from vanes of an otherwise perfect attenuator. The result is found to contain three complex constants that can be determined experimentally, with this expression one can, when necessary, take reflections into account analytically by determining the constants appropriate to the attenuator under consideration. This procedure is useful, e.g., when using a combination of an attenuator and a movable short-circuit as a variable impedance standard [3]; or, when determining "mismatch error" in a transmission system in which the generator or load are not matched to the line [4], [5].

Consider the cascade of three networks shown in Fig. 1. Since each circular waveguide supports two mutually-orthogonal dominant modes, the transitions and circular section are three- and four-ports, respectively. The (symmetrical) scattering matrices of these networks are thus of the form

$$S_a = \begin{bmatrix} S'_{55} & S'_{56} & S'_{57} \\ & S'_{66} & S'_{67} \\ & & S'_{77} \end{bmatrix} \tag{1}$$

$$S_b = \begin{bmatrix} S'_{11} & S'_{12} & S'_{13} & S'_{14} \\ & S'_{22} & S'_{23} & S'_{24} \\ & & S'_{33} & S'_{34} \\ & & & S'_{44} \end{bmatrix} \tag{2}$$

$$S'_c = \begin{bmatrix} S'_{88} & S'_{89} & S'_{8,10} \\ & S'_{99} & S'_{9,10} \\ & & S'_{10,10} \end{bmatrix} \tag{3}$$

where coefficients subscripts are defined by the polarization directions shown in Fig. 1. Absorbing vanes are assumed to lie in the horizontal plane of each transition and in the 2–4 plane of the rotating section.

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By applying the appropriate coordinate transformation, one can solve for the overall scattering matrix

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12} & S_{22} \end{bmatrix} \tag{4}$$

of the cascade. To simplify this calculation, we assume that S_{56}', S_{13}', and S_{89}' are of the form

$$S_{ij}' = 1 - \delta_{ij} \tag{5}$$

while all other terms are of the form

$$S_{ij}' = \delta_{ij} \tag{6}$$

with δ_{ij} small. Retaining only terms to first order in δ_{ij} yields an equation of the form

$$S_{11} = A_{11} + B_{11}\sin^2\theta + C_{11}\sin^2(2\theta) + D_{11}\sin(2\theta) + E_{11}\sin(4\theta) \tag{7}$$

with a similar result for S_{22}. The constants in (7) are complicated functions of the various δ_{ij}'s; and the vane angle θ is related to the attenuator setting in dB by

$$\cos^2\theta = 10^{-\mathrm{dB}/20}. \tag{8}$$

Equation (7) can be simplified by assuming further that vanes absorb all transmitted waves polarized parallel to them and that no cross-coupling occurs between spatially orthogonal modes. That is

$$\left.\begin{aligned} S'_{57} = S'_{67} = S'_{8,10} = S'_{9,10} = 0 \\ S'_{12} = S'_{14} = S'_{23} = S_{34}' = 0 \\ S'_{24} = 0 \end{aligned}\right\}. \tag{9}$$

The overall reflection coefficient S_{11} is then of the form

$$S_{11} = A_{11} + B_{11}\sin^2\theta + C_{11}\sin^2(2\theta) \tag{10}$$

with a similar result for S_{22}. Under the same approximations which lead to (10), the transmission coefficient is found to be

$$S_{12} = A_{12}\cos^2\theta \tag{11}$$

where A_{12} is a complex constant with magnitude less than unity. Thus, besides causing variable terminal reflections, slightly reflecting vanes will also introduce a fixed insertion loss and a fixed phase shift. To first order they do not, however, cause variable errors in attenuation, nor do they cause phase shift that varies with attenuator setting. Equation (11) has been found to be well satisfied in measurements of attenuation [5] and phase shift [6] of actual rotary-vane attenuators.

In order to check the validity of (10), the magnitude $|S_{11}|$ and $|S_{22}|$ of several commercially-made X- and K-band attenuators were measured with reflectometers that had been tuned by the procedure of Engen and Beatty [7]. In each measurement, the opposite port of the attenuator was terminated in a matched load that had been tuned to eliminate reflections. Figures 2 and 3 show typical results. Points represent experimental values while curves were calculated from (10) using the constants given in Table I. These constants were determined by fitting the results to (10) at five experimental points. Since the magnitude of S_{ii} depends on A_{ii}, B_{ii}, and C_{ii} only to within a common arbitrary phase angle, A_{ii} was assumed real. One notes excellent agreement between theory and experiment over the entire attenuation range. Although

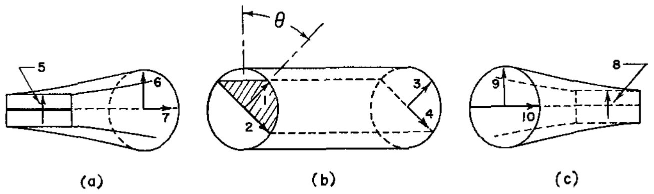

Fig. 1. Rotary-vane attenuator represented by cascade of three networks. Transitions are three-ports while the circular section is a four-port. Absorbing vanes lie in horizontal plane of transitions and in the 2–4 plane of circular section.

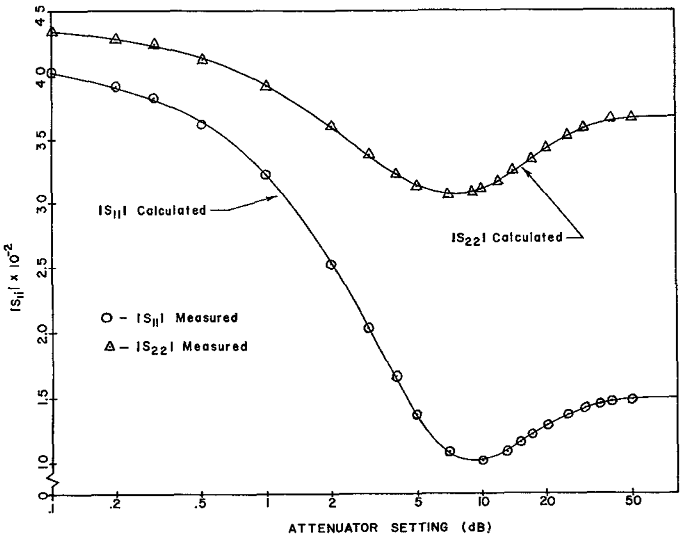

Fig. 2. Measured and calculated magnitudes of reflection coefficients of FXR X164A X-band attenuator.

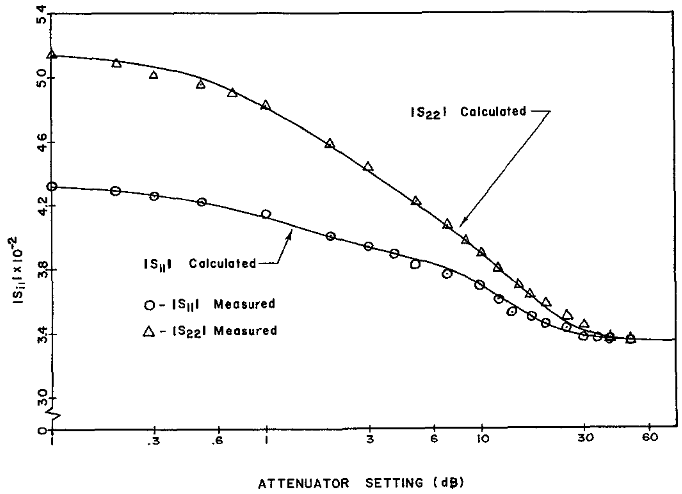

Fig. 3. Measured and calculated magnitudes of reflection coefficients of FXR K164AF K-band attenuator.

TABLE I
ATTENUATOR CONSTANTS

	FXR X164A X-Band Attenuator		FXR K164AF K-Band Attenuator	
	S_{11}	S_{22}	S_{11}	S_{22}
A_{ii}	0.0412	0.0439	0.0436	0.0520
B_{ii}	0.0560 $e^{-3.16}$	0.0679 $e^{-j3.66}$	0.0580 $e^{-j3.75}$	0.0334 $e^{-j2.46}$
C_{ii}	0.0125 $e^{-j4.04}$	0.00927 $e^{-j5.06}$	0.0135 $e^{-j5.10}$	0.0106 $e^{-j1.88}$

the phase of S_{ii} was not measured, it is believed unlikely that it would not similarly agree with theory.

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Temperature Dependence of Composite Coaxial Resonators

Under certain circumstances, it is necessary to construct a composite coaxial resonator using inner and outer conductors having different thermal expansion coefficients. This may be required for mechanical reasons or as a means of adjusting the resonator temperature stability. This correspondence describes an analysis of such a resonator as well as some experimental results.

The resonant frequency of a quarter-wavelength capacitively loaded line is given by

$$\frac{10^{-12}}{\omega CZ} = \tan 2\pi L/\lambda \quad (1)$$

where

ω = resonant frequency, rad/s
C = loading capacitance, pF
Z = impedance of coaxial line, ohms
L = length of inner conductor, cm
λ = wavelength at resonance, cm.

Equation (1) is transcendental and can be solved by graphical means[1] if the loading capacitance is known, by letting

$$Y_{11} = \tan 2\pi L/\lambda \quad (2)$$

and

$$Y_{12} = \frac{\lambda\epsilon}{2\pi vCZ} \quad (3)$$

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